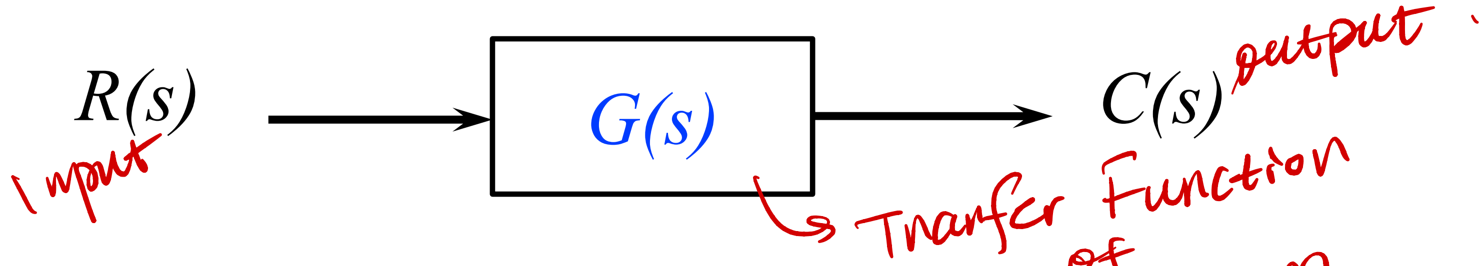
GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

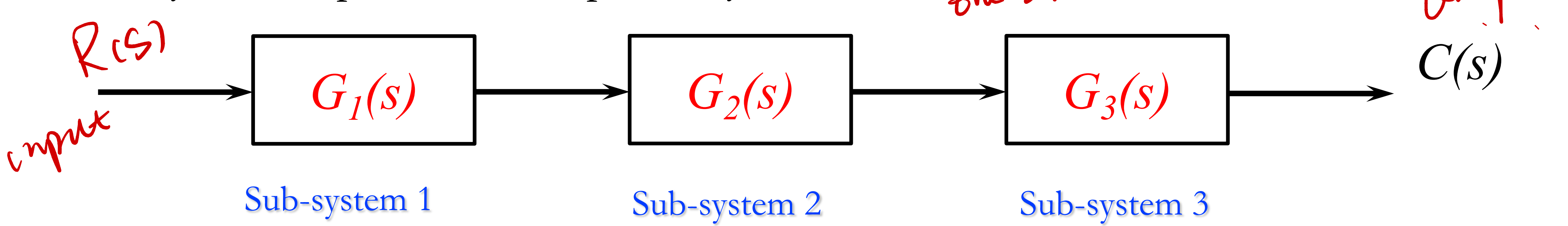
Graphical Models

Block Diagram

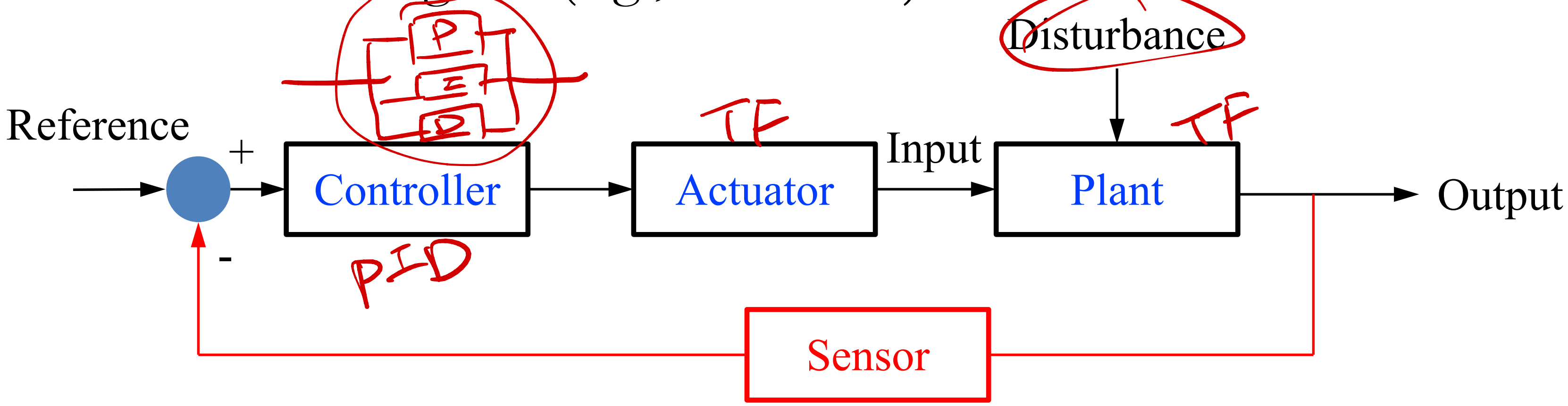
Block Diagram



Systems usually are composed of multiple subsystems:

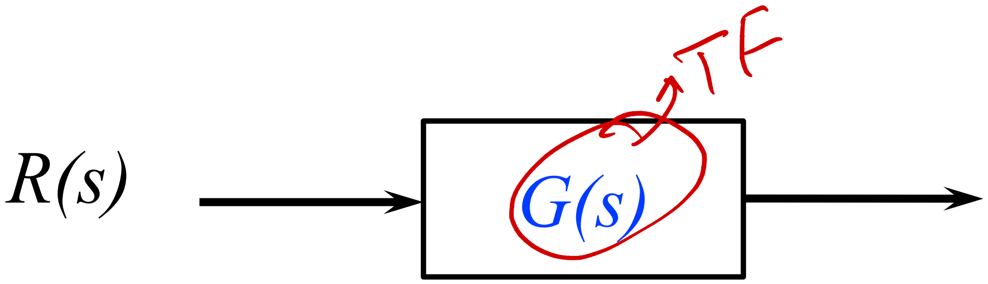


More complex control block diagram (e.g., Feedback)

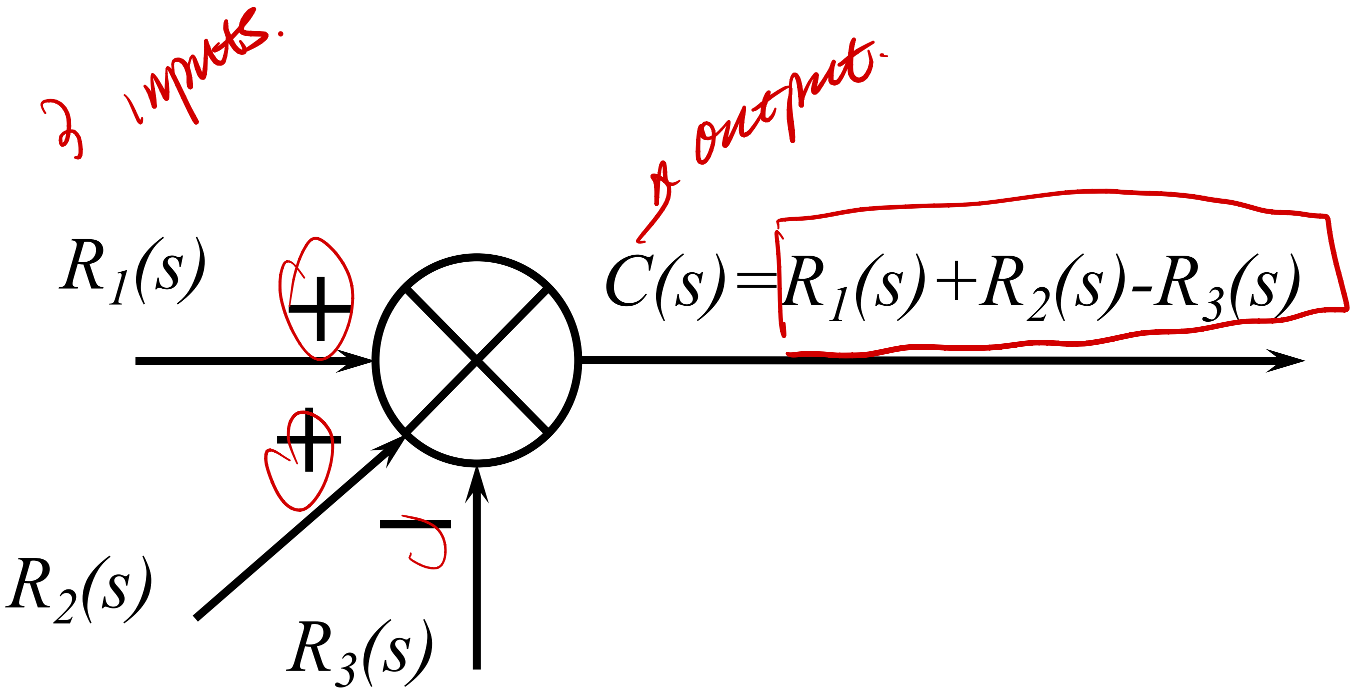


Basic Components of Block Diagrams

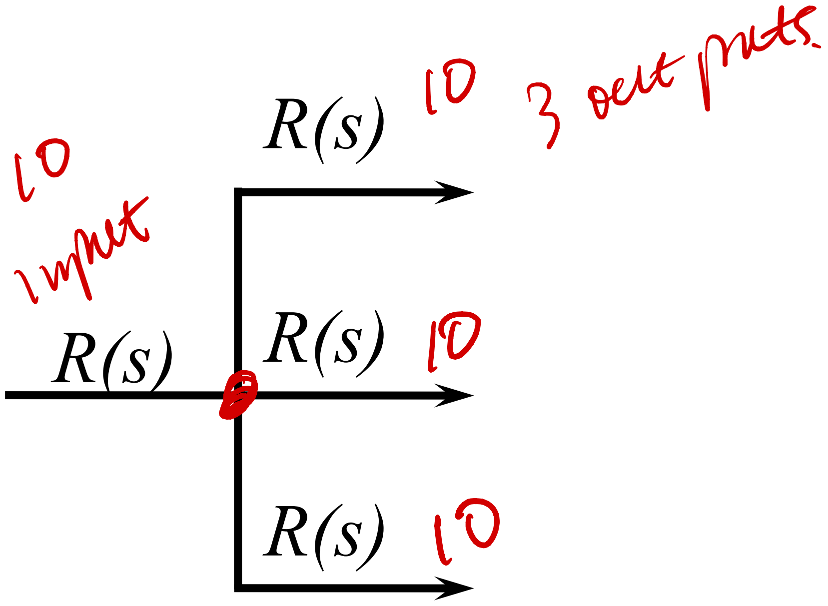
Signals



System

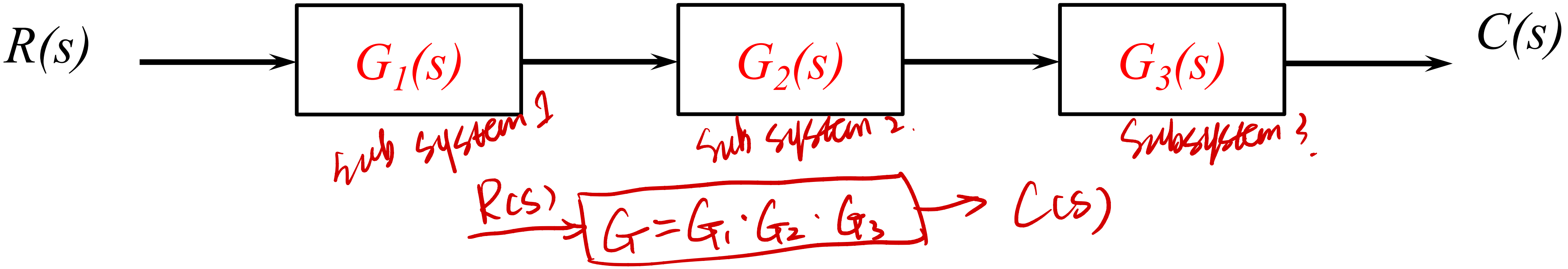


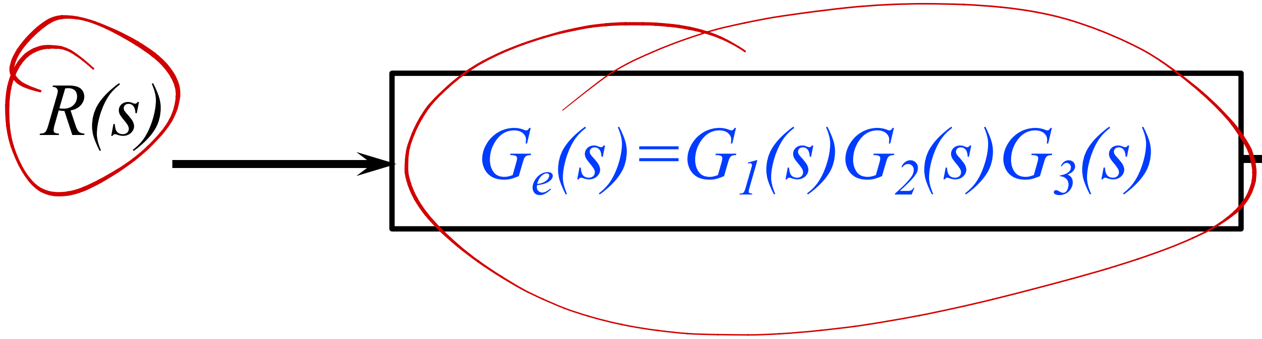
Summing Junction



Pickoff Point

Cascade Form





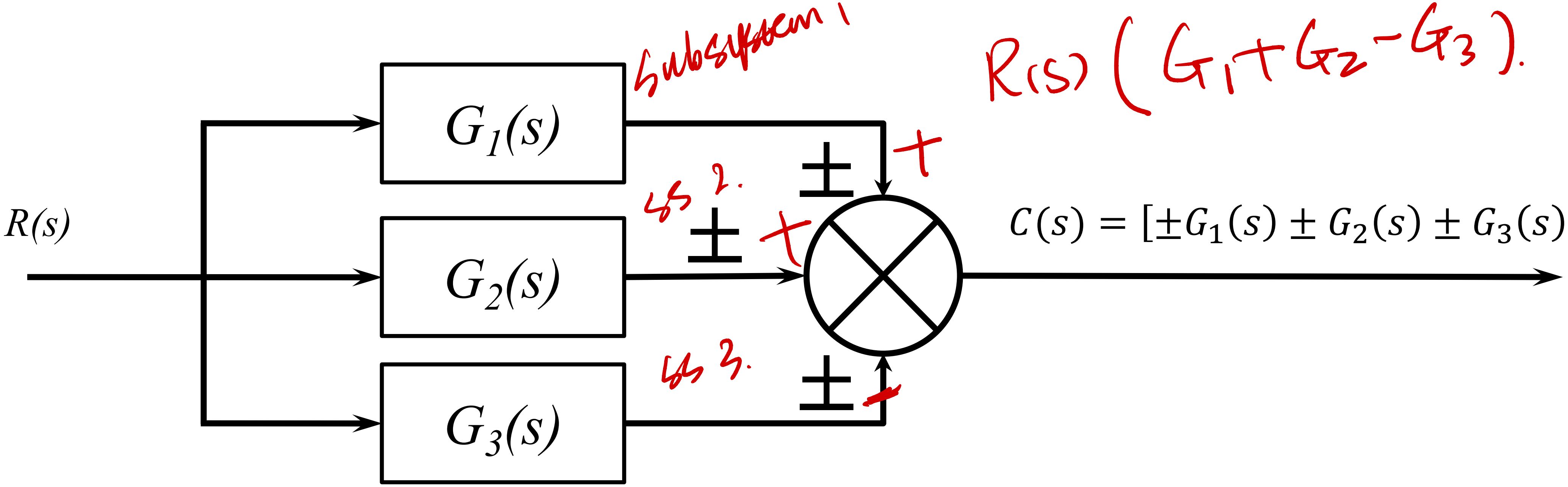
$$C(s)=[G_1(s)\ G_2(s)G_3(s)]R(s)$$

$G_e \cdot R$

TF of system

input

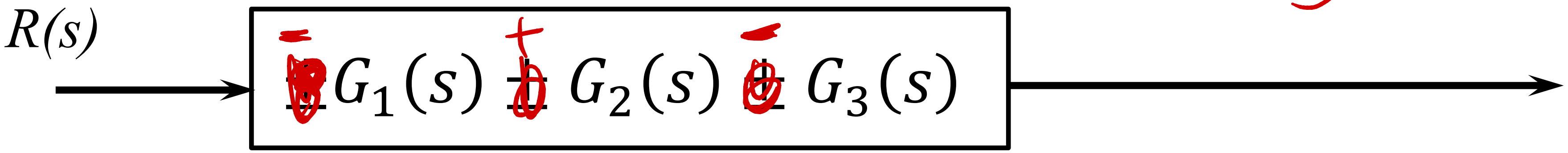
Parallel Form



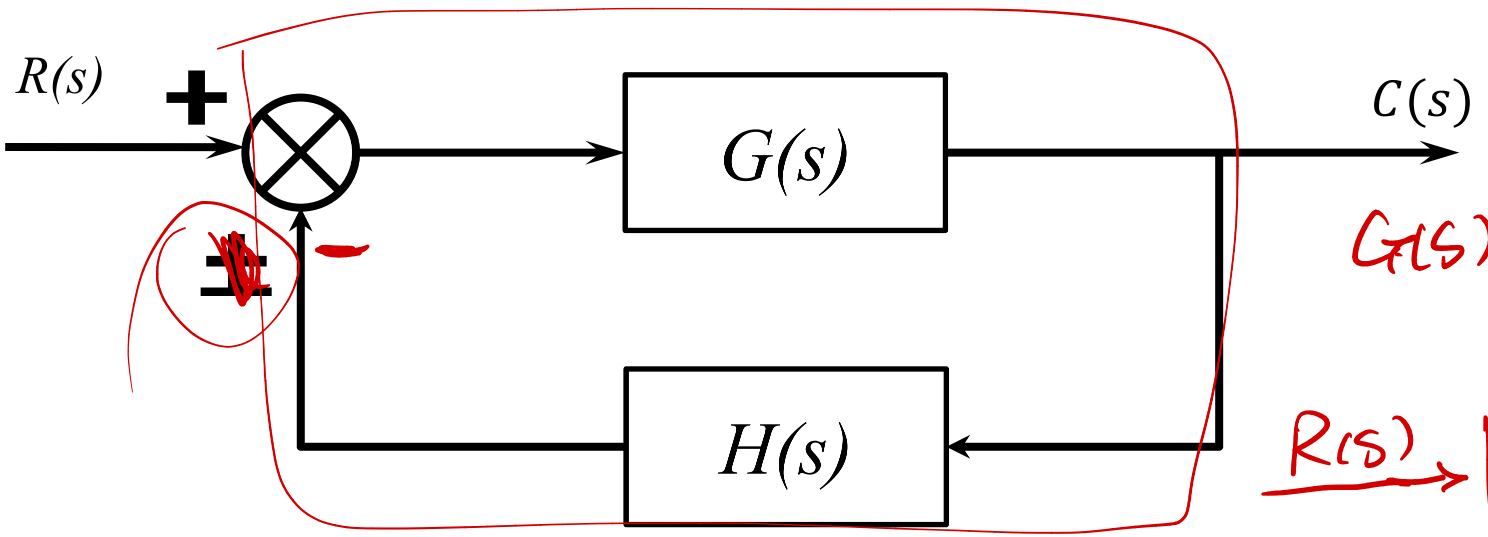
$$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$$

$R(s)(G_1 + G_2 - G_3).$

$C(s) = R(s)(-G_1 + G_2 - G_3)$



Feedback Form: Eliminating a Feedback Loop



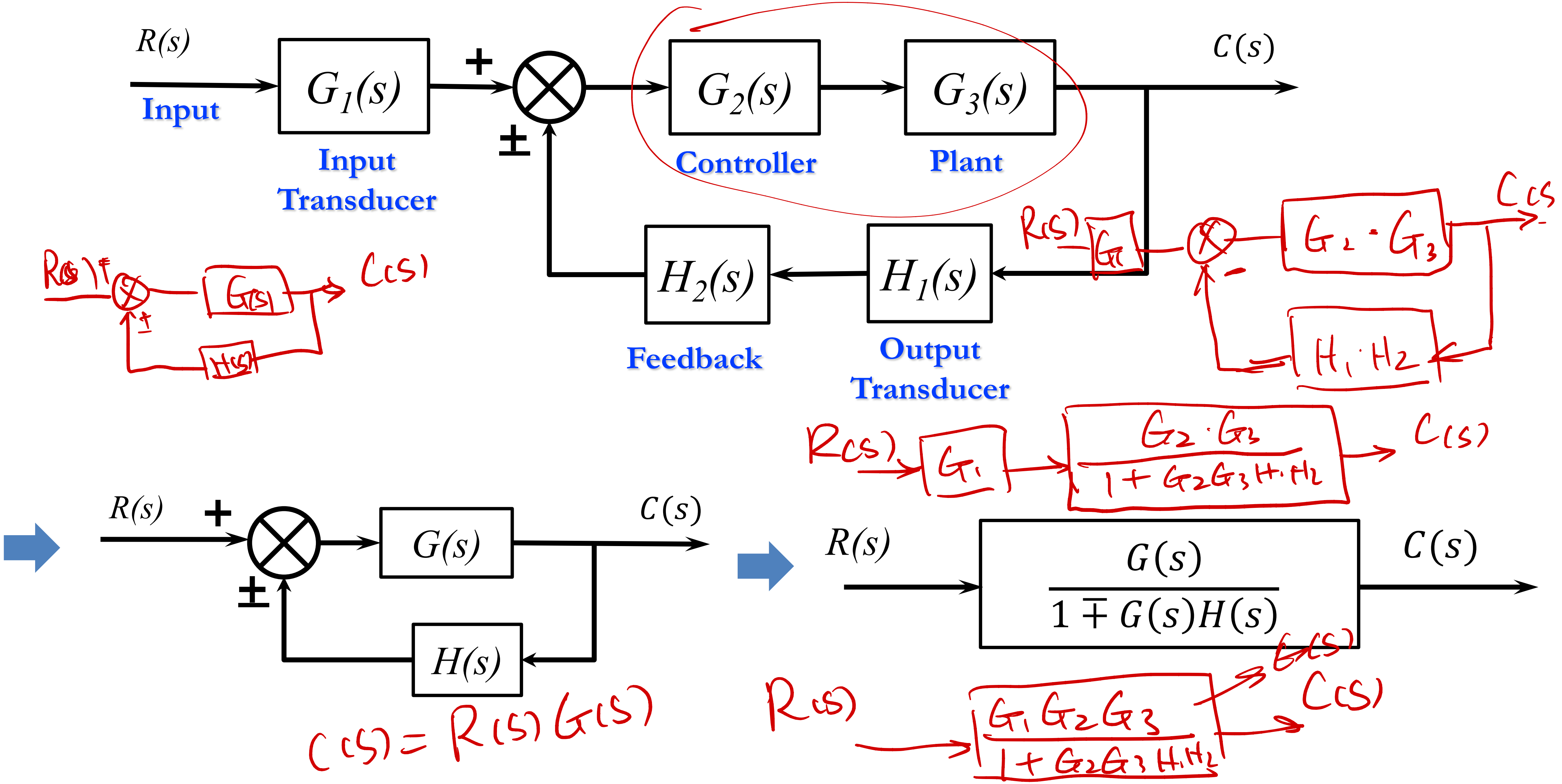
Feedback (Sensor)

$$G(s) = \frac{G(s)}{1 + G(s)H(s)}$$

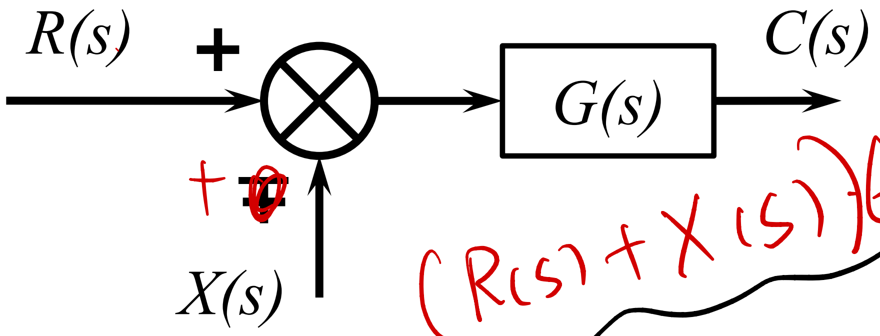
$$R(s) \rightarrow \boxed{\frac{G(s)}{1+G(s)H(s)}} \rightarrow C(s)$$

$$R(s) \rightarrow \boxed{\frac{G(s)}{1 \mp G(s)H(s)}} \rightarrow C(s)$$

Moving Blocks to Create Familiar Forms

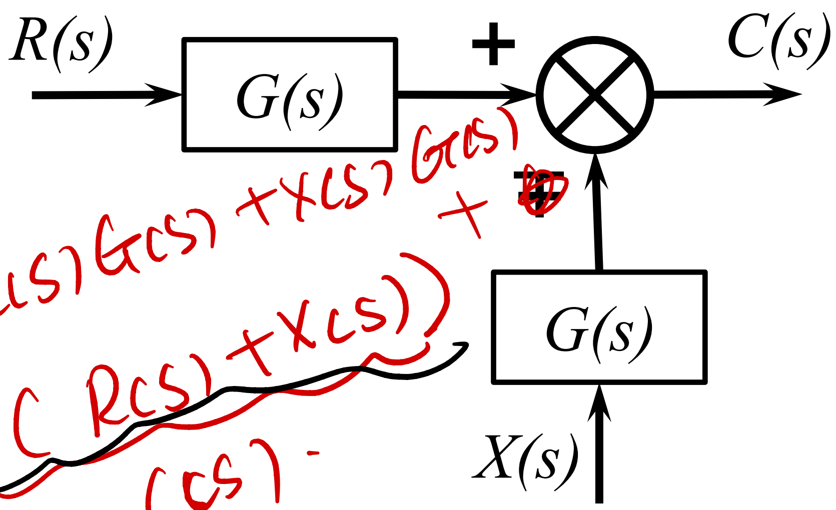


Moving a Summing Junction



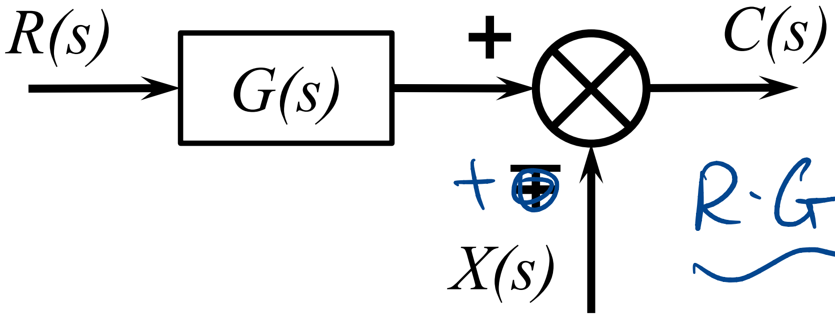
$\equiv$

$(R(s)+X(s))G(s) = C(s).$



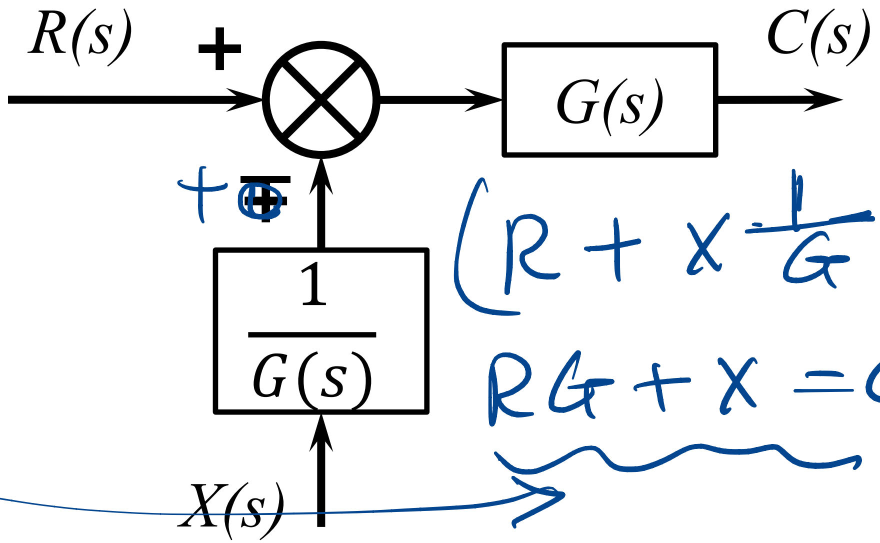
$R(s)G(s)+X(s)G(s)$

$G(s)(R(s)+X(s)) = C(s).$



$\equiv$

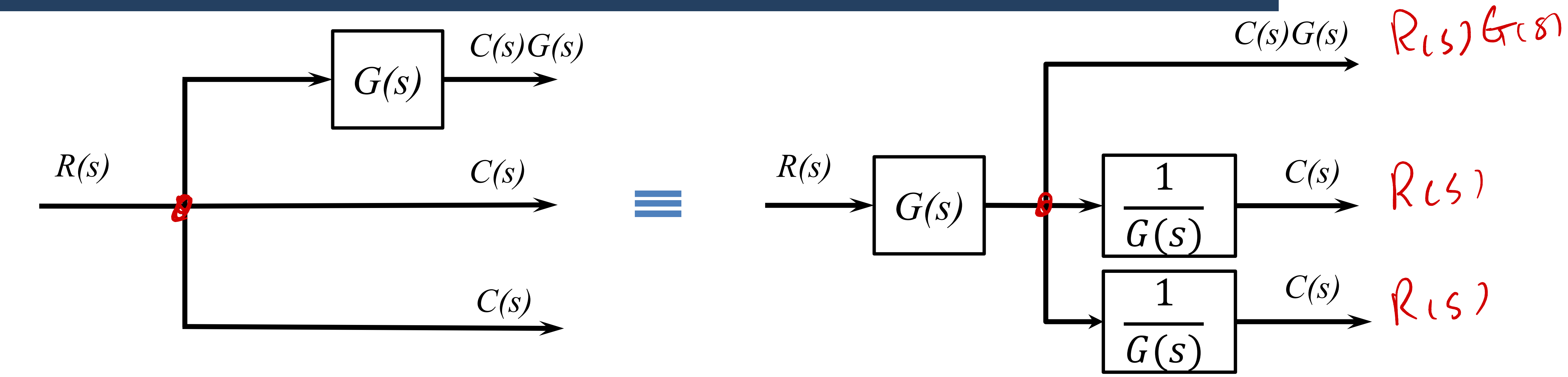
$R \cdot G + X = C$

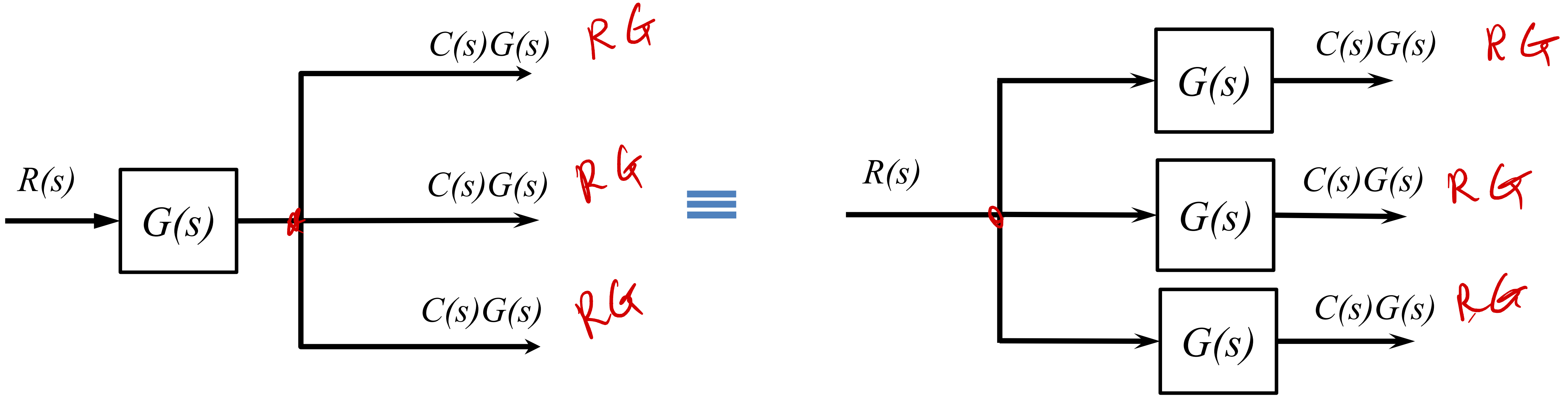


$(R + X \cdot \frac{1}{G}) \cdot G = C$

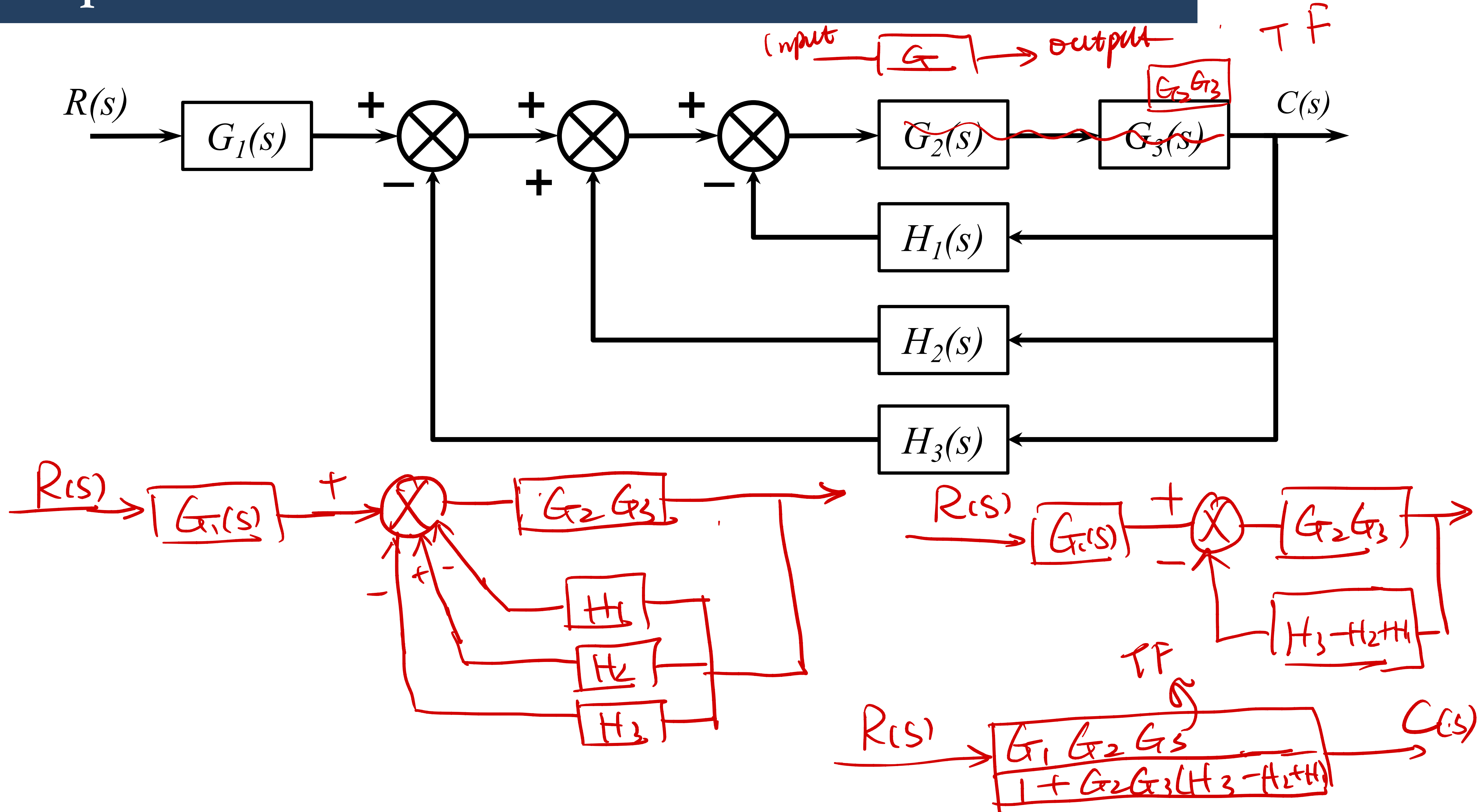
$RG + X = C$

Moving a Pickoff Point



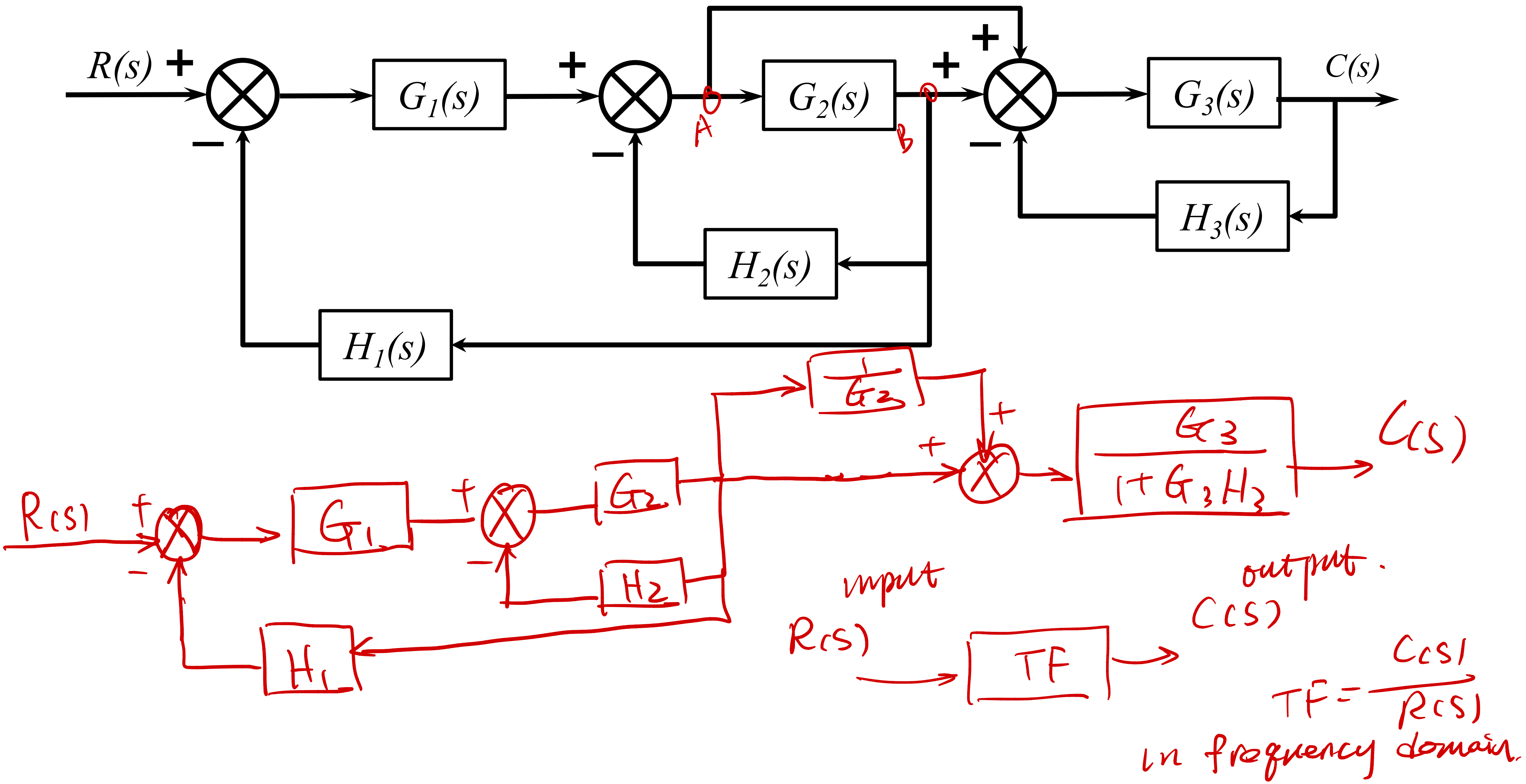


Example 1


input
G
output
TF
G_2G_3
$R(s)$
$G_1(s)$
+
−
+
+
+
−
$G_2(s)$
$G_3(s)$
$C(s)$
$H_1(s)$
$H_2(s)$
$H_3(s)$
$R(s)$
$G_1(s)$
G_2G_3
H_1
H_2
H_3
$R(s)$
$G_1(s)$
G_2G_3
$H_3-H_2+H_1$
TF
$R(s)$
$\frac{G_1G_2G_3}{1+G_2G_3(H_3-H_2+H_1)}$
$C(s)$

Example 1 (Cont'd)

Example 2

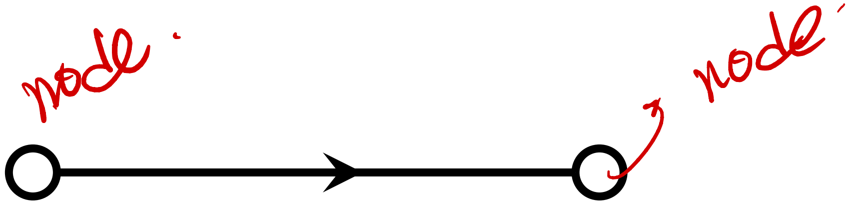

R(s)
+
−
G1(s)
+
−
A
G2(s)
B
+
+
−
G3(s)
C(s)
H3(s)
H2(s)
H1(s)
R(s)
+
−
G1
+
−
G2
H2
H1
1/G2
+
+
G3 / (1+G3H3)
C(s)
input
R(s)
TF
output
C(s)
TF = C(s)/R(s)
in frequency domain

Example 2 (Cont'd)

Signal Flow Graph

Definition

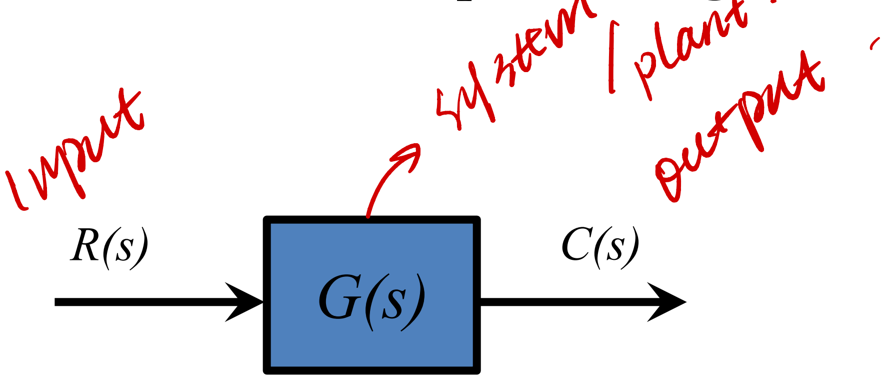
A system is represented by a line with an arrow showing the direction of signal flow through the system.

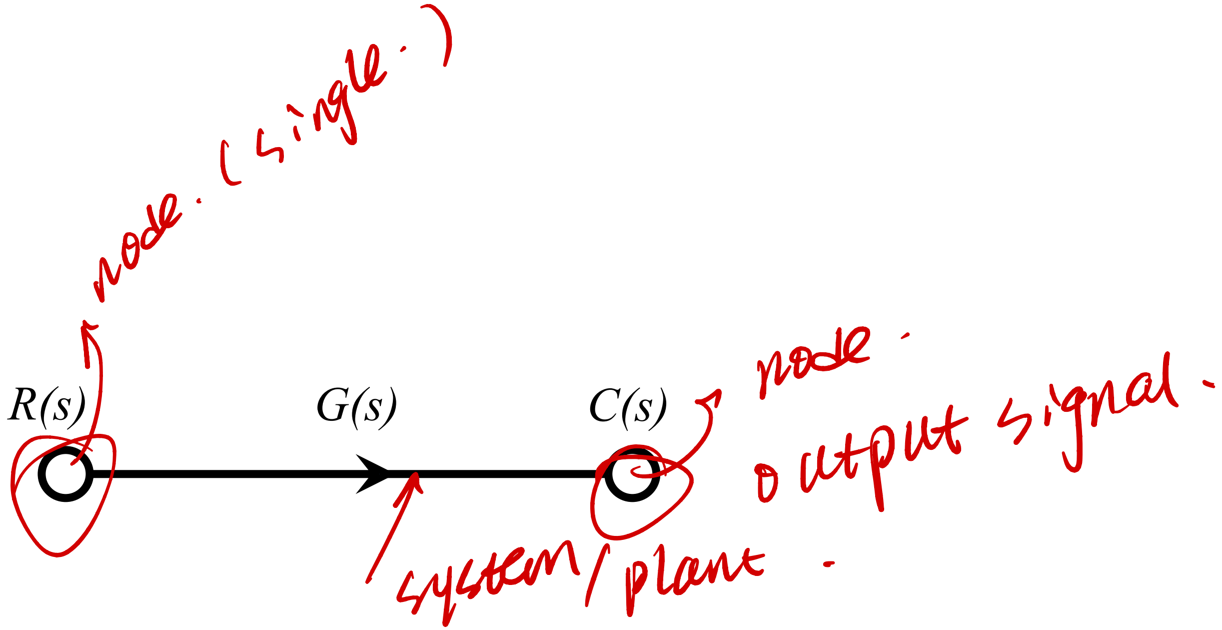


A signal-flow graph consists only **branches** and **nodes**:

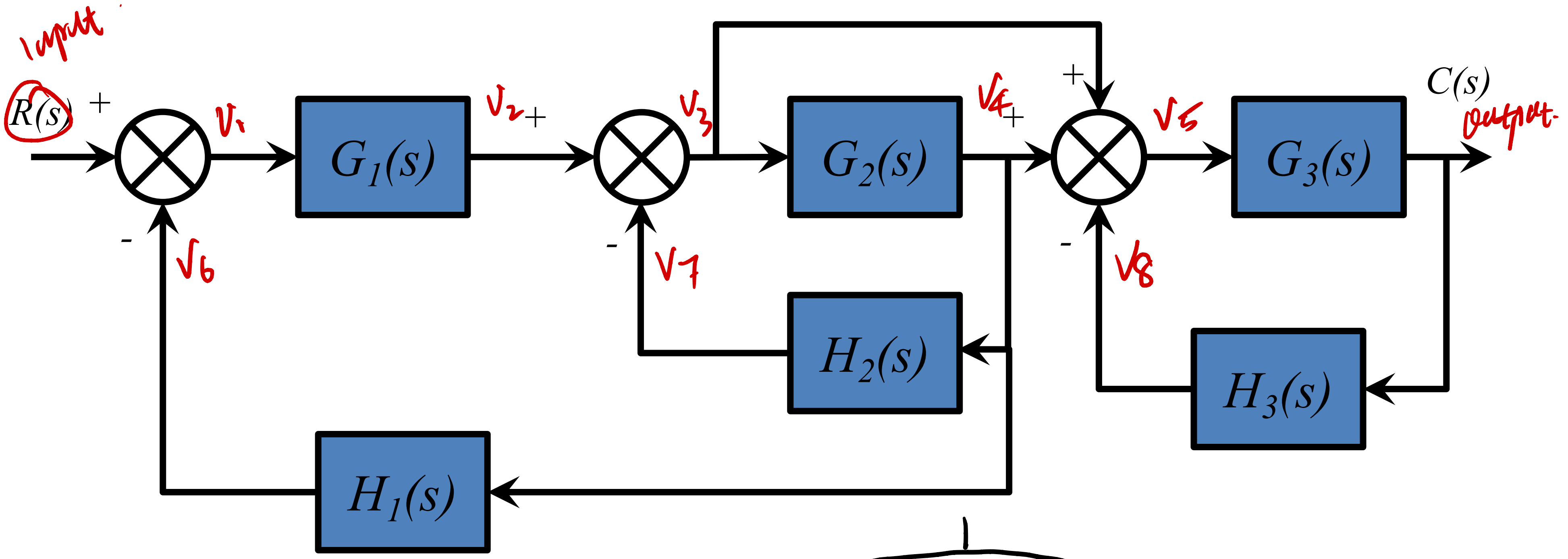
Branches: represent **systems**

Nodes: represent **signals**

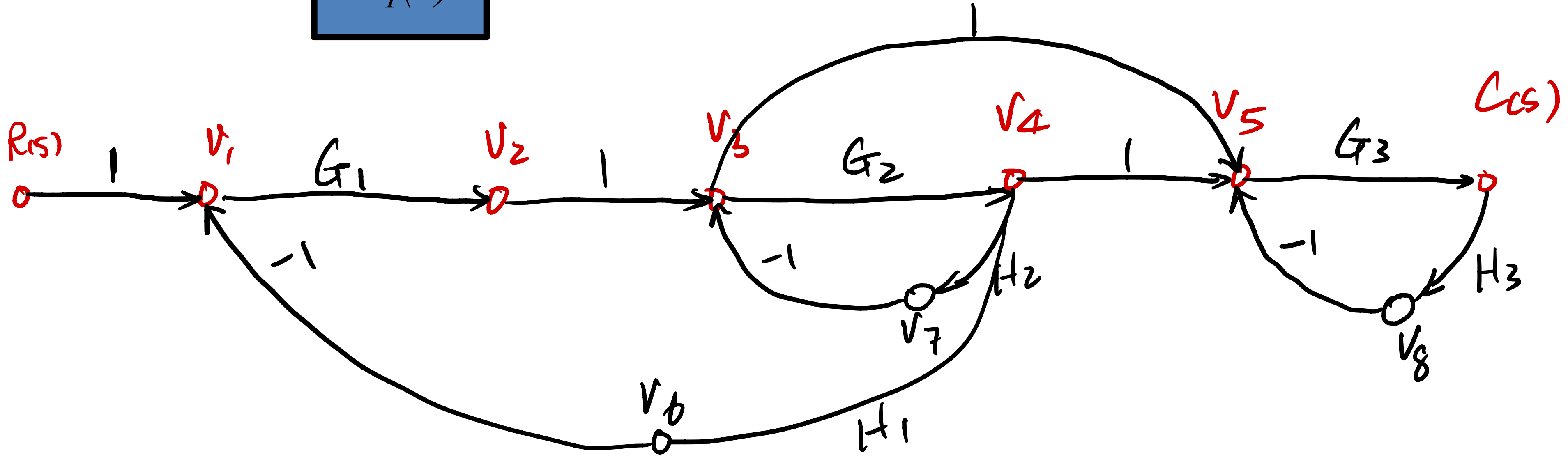




Example



Signal => node.
system => line.



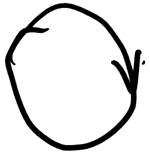
output

$$TF = \frac{C(s)}{R(s)}$$

input

Analytical model

Mason’s Rule: Definition



Loop Gain:

The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

Forward-path Gain:

The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

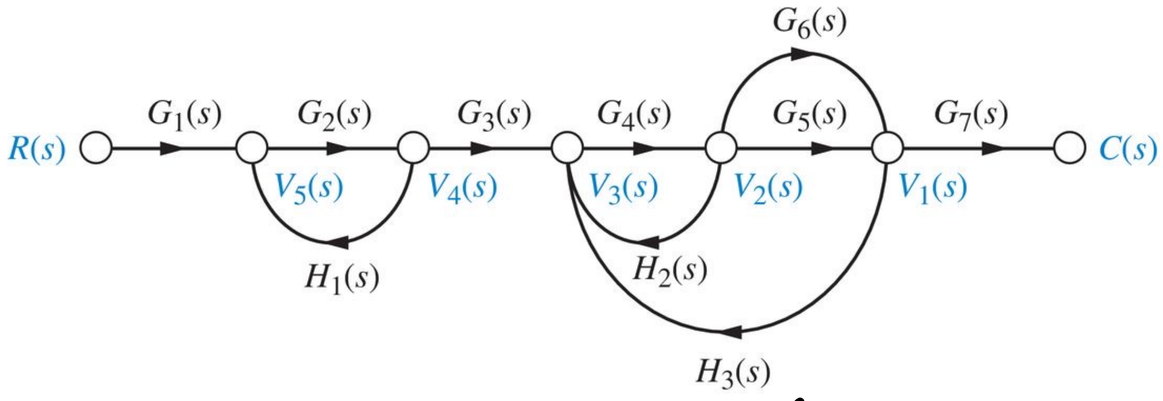
Non-touching Loops:

Loops that do not have any nodes in common.

Non-Touching-Loop Gain:

The product of loop gains from non-touching loops taken two, three four, or more at a time

Example



$$TF = \frac{C(s)}{R(s)}$$

Loop Gain:

$G_2 \cdot H_1$

$G_4 \cdot H_2$

$G_4 \cdot G_5 \cdot H_3$

$G_4 \cdot G_6 \cdot H_3.$

Forward-path Gain:

$G_1 G_2 G_3 G_4 G_5 G_7$

$G_1 G_2 G_3 G_4 G_6 G_7$

Non-touching Loops:

$G_2 H_1$ $G_4 H_2$ $G_4 G_5 H_3$ $G_4 G_6 H_3.$

Non-Touching-Loop Gain: $[G_2 H_1][G_4 H_2]$ $[G_2 H_1][G_4 G_5 H_3]$

$[G_2 H_1][G_4 G_6 H_3]$

Mason's Rule

output

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

input

k = number of forward paths

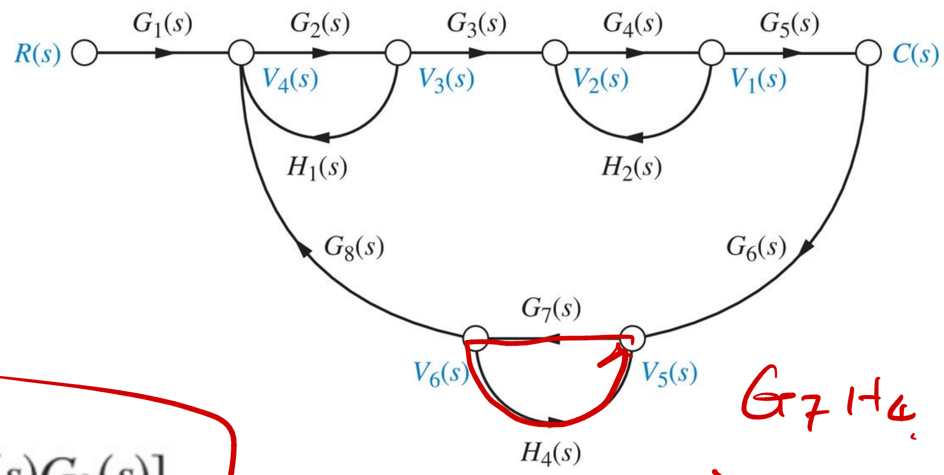
T_k = the *k*th forward-path gain

Δ = 1- Σ loop gains + Σ non-touching loop gains taken two at a time – Σ non-touching loop grains taken three at a time + Σ non-touching loop gains taken four at a time …

Δ_k = Δ – Σ loop gain terms in Δ that touch the *k*th forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the *k*th forward path.

Example

Find the transfer function, $C(s)/R(s)$ for the signal-flow-graph:



$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta}$$

$$\begin{aligned}\Delta = 1 &- [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &\qquad + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &\qquad + G_4(s)H_2(s)G_7(s)H_4(s)] \\ &- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

non touching gain (taken three).
G2H1 G4H2 G7H4.

Forward path gain: G1 G2 G3 G4 G5
Loop gain - G2H1 G4H2 G7H4 G2G3G4G5G6G7G8
non-touching loop gain (take two): G2H1 G4H2 (loop 1 2) G2H1 G7H4
G4H2 G7H4

Signal-Flow Graphs of State Equations

Consider the following state and output equations:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \end{bmatrix}
$$

$$
\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}
$$

$$
\begin{cases}
\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r \\
\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r \\
\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r \\
y = -4x_1 + 6x_2 + 9x_3
\end{cases}
$$

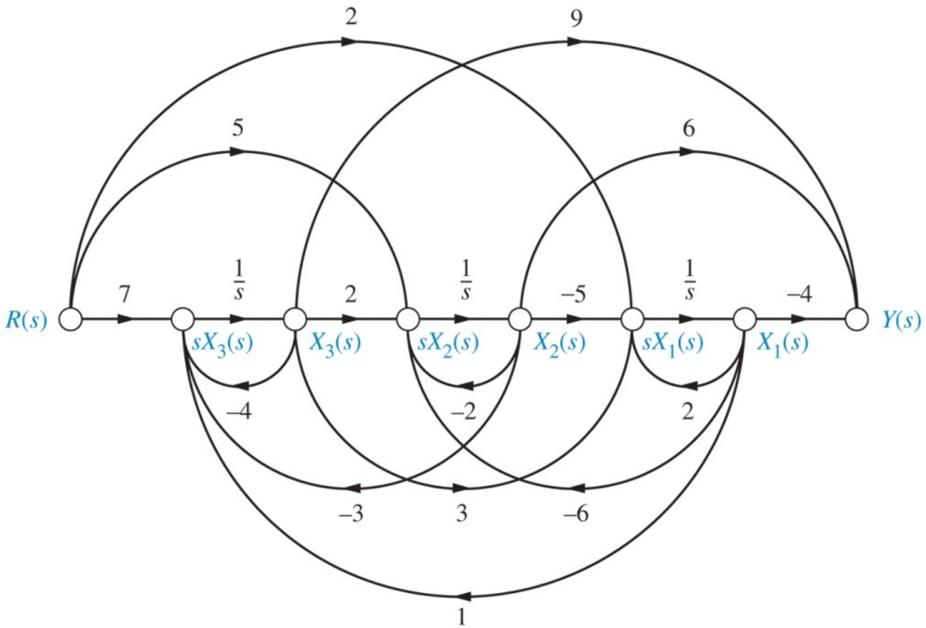
$\dot{x}_2 \xrightarrow{\frac{1}{s}} x_2$

$\dot{x}_3 \xrightarrow{\frac{1}{s}} x_3$

system

$\dot{x}_1 \xrightarrow{\frac{1}{s}} x_1$

where r is the input, y is the output, x_1, x_2 and x_3 are the state variables, please draw its signal-flow graph.



The End!!